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CASCADE PROCESSES AND FRACTALS IN TURBULENCE

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The idea of homogeneity is now giving way to the less restrictive idea of fractal self-similarity (see, for example, [1-4]). Cascade models, which have been used successfully for studying homogeneous turbulence, could also be useful for studying fractal turbulence. On the other hand, these cascade models can be improved by taking into account the fractal structure of real turbulence. A great deal of experimental material has now been accumulated. This material needs to be organized and interpreted. In the present paper we examine some aspects of cascade processes taking into account the fractal structure of turbulence. First, we establish a relation between the form of the energy spectral density in the scaling interval and the fractal dimension of the surface of the hydrodynamic fields. This relation is important, in particular, for atmospheric turbulence and is confirmed by direct observations of atmospheric hydrodynamic fields, performed by different authors. An analogous investigation was also performed for two-dimensional turbulence, the computational results for which are confirmed by comparing with oceanographic computational data. Second, a relation between the constant in the Kolmogorov–Obukhov spectral law and the intermittency coefficient is established by taking into account the fractal structure.

Suppose that when turbulence arises it has a patchy character, i.e., the nonturbulent region contains separate subregions occupied with turbulent fluid [5] (criteria for distinguishing between the subregions are given, for example, in [6]). Since the fluid particles in the turbulent liquid strive to move away from each other [7, 8], one would expect that in time these regions will expand on the average. Moreover, this property of fluid particles in a turbulent liquid should, in general, cause the turbulent part of the liquid to strive constantly to increase the total area of the boundary separating it from the nonturbulent fluid. Is this process unbounded or can it saturate? If a self-similar situation is established, then the total area of the surface separating the turbulent liquid from the nonturbulent liquid will approach infinity, and this surface will become a fractal with fractal dimension $D_\sigma > 2$ (in three-dimensional space).

We introduce the probability density $\rho(\ell)$ for encountering a turbulent subregion with characteristic size ℓ. By definition of the probability density the total area separating the turbulent and nonturbulent regions in the interval of self-similarity is given by

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$$S \sim \int_{\eta}^{L} \rho(l)\, l^2\, dl, \tag{1}$$

where L and η are the upper and lower limits of the self-similarity interval. If in this interval the distribution $\rho(\ell)$ follows a power law, i.e.,

$$\rho(l) \sim l^{-x}, \tag{2}$$

then we obtain from Eq. (1)

$$S \sim \frac{L^{3-x}}{(3-x)}\left[1-\left(\frac{\eta}{L}\right)^{3-x}\right]. \tag{3}$$

Since we are interested only in the exponents and dimensional considerations are not used, we employ an approximate form of the formulas, where possible, in order to simplify the expressions. This will not affect the accuracy with which the exponents are determined.

Usually $L \gg \eta$. For this reason, for $x < 3$

$$S \sim L^{3-x}/(3-x), \tag{4}$$

and for $x > 3$

$$S \sim (L/\eta)^{x-3}/(x-3), \tag{5}$$

i.e., for $x > 3$ in the self-similar asymptotic range, when $\eta/L \to 0$, $S \to \infty$; in other words, the total area of the surface separating the turbulent and nonturbulent fluids in the self-similarity interval for $x > 3$ will become fractal. For a distribution of turbulent regions over scales of the form (2) with $x > 3$ a fractal surface structure will form (in the self-similarity interval).

It is not difficult to relate its quantitative characteristic, i.e., the fractal dimension D_σ of the interface, with the distribution (2). On the one hand, the asymptotic behavior of the fractal surfaces is known

$$S \sim (L/\eta)^{D_\sigma - 2} \tag{6}$$

(in three-dimensional space), while on the other hand it follows from Eq. (5) that

$$S \sim (L/\eta)^{x-3}.$$

Therefore,

$$x = D_\sigma + 1, \tag{7}$$

i.e., in the self-similarity interval the distribution of turbulent regions over their size scales has the form

$$\rho(l) \sim l^{-(D_\sigma+1)}. \tag{8}$$

By definition of the probability density $\rho(\ell)$ the total volume occupied by the turbulent subregions whose characteristic dimensions fall within the self-similarity interval is given by the formula

$$V = \int_{\eta}^{L} \rho(l)\, l^3\, dl. \tag{9}$$

Substituting the expression (8) into Eq. (9) we obtain

$$V \sim L^{3-D_\sigma}\left[1-(\eta/L)^{3-D_\sigma}\right]. \tag{10}$$

If in the self-similarity interval the total energy of the turbulent liquid W is additive over the turbulent regions, then $W \sim V$ and hence

$$W \sim L^{3-D_\sigma}\left[1-(\eta/L)^{3-D_\sigma}\right]. \tag{11}$$

On the other hand, if in the self-similarity interval the spectral density of the energy is given by

$$E(k) \sim k^{-\alpha}, \tag{12}$$

then

$$W \sim \int_{L^{-1}}^{\eta^{-1}} E(k)\, dk \sim L^{\alpha-1}\left[1-(\eta/L)^{\alpha-1}\right]. \tag{13}$$

Then we find from the relations (11) and (12)

$$4-\alpha=D_{\sigma}. \quad (14)$$

This formula relates the scaling exponent in the spectral density of the energy to the fractal dimension of the interface of the turbulent and nonturbulent fluids in the self-similarity interval.

The most familiar value of α is 5/3 (Kolmogorov–Obukhov law [7, 8]), for which we obtain from Eq. (14)

$$D_{\sigma}=7/3. \quad (15)$$

The fractal dimension of cloud surfaces has been measured in a number of experiments and values right up to ~10^6 km^2 (with very large Reynolds numbers [9]) have been obtained. In different experiments, performed with the help of a satellite in the infrared region of the spectrum and radar observations, values $D_\sigma \approx 7/3$ have been obtained. This can serve as a confirmation of what we have said above as well as evidence that the Kolmogorov–Obukhov law (α = 5/3) is satisfied in atmospheric processes [see Eq. (14)].

Two-dimensional turbulence is under intensive study [10, 11]. For it the formulas relating D and α are somewhat different. If the turbulence energy is additive, then we find, analogously to the formula (14),

$$D_{\sigma}=3-\alpha. \quad (16)$$

Then with α = 5/3 for two-dimensional turbulence the fractal dimension of the interface between the turbulent and nonturbulent fluids is $D_\sigma = 4/3$. Such a fractal dimension has been observed in experiments performed in the Kuroshio Current [12].* In two-dimensional turbulence the situation when the enstrophy (mean-square vorticity; in the two-dimensional case the enstrophy is also an integral of the motion [10]), and not the kinetic energy is additive is also of great interest.

The part of the enstrophy concentrated in scales falling within the self-similarity interval is

$$\bar{\Omega}_{*} \sim \int_{L^{-1}}^{\eta^{-1}} k^{2} E(k)\, dk \sim L^{\alpha-3}\left[1-(\eta/L)^{\alpha-3}\right]. \quad (17)$$

If in this interval the enstrophy is additive, then

$$\bar{\Omega}_{*} \sim \int_{\eta}^{L} \rho(l)\, l^{3}\, dl \sim L^{2-D_{\sigma}}\left[1-(\eta/L)^{2-D_{\sigma}}\right]. \quad (18)$$

Then we obtain from Eqs. (17) and (18)

$$D_{\sigma}=5-\alpha. \quad (19)$$

In the two-dimensional case $1 \leqslant D_{\sigma} \leqslant 2$; as turbulence develops one can expect that D_σ will increase from 1 to 2, i.e., from maximum uniformity to maximum nonuniformity. Then we conclude from Eq. (19) that as the two-dimensional turbulence develops the value of α will decrease from 4 to 3 as D_σ increases from 1 to 2. In this sense the numerical experiment described in [11] is interesting. The evolution of the exponent in this experiment is shown in Fig. 1, which is taken from [11]. Consider the self-similar process of cascade fragmentation of eddies [8, 13]. Some laminar, hydrodynamically unstable [8] eddy, formed as a result of fragmentation of a larger eddy (at the intermediate stage of this process), will itself decay into still smaller eddies. The process of scale reduction continues q times [13]. Some of the eddies formed as a result of such fragmentation become turbulent (for this reason, they are more stable and are not subject to decay). The other laminar eddies formed are unstable and decay into smaller eddies. For them, in turn, the entire process is repeated until the scale of the eddies formed is sufficiently small that the eddies are stabilized by molecular viscosity. This scheme roughly takes into account intermittency, since turbulent and laminar eddies are present at any stage of a self-similar cascade (the specific mechanism by which the eddies become turbulent as they decay is not important for what follows).

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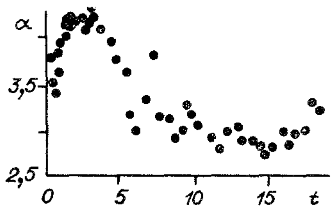

Fig. 1

If the multiplicity of the fragmentation scale is equal to q, then after the first fragmentation approximately q^d secondary eddies are formed (d is the dimension of the space in which the process occurs). Of them, γq^d eddies are turbulent and stable, while $(1-\gamma)q^d$ eddies are laminar and unstable. The unstable eddies decay with the same multiplicity of the fragmentation scale q and with the same intermittency constant γ. In the process, there now form $\gamma q^d(q^d - \gamma q^d)$ turbulent eddies of the next order, analogously $\gamma q^d(q^d - \gamma q^d)^2$ turbulent eddies in the next order in the cascade, etc. In general, to the scale $\ell_n/L = q^{-n}$ (L is the initial length scale) there correspond $\gamma q^d(q^d - \gamma q^d)^{n-1}$ turbulent eddies. Thus there is established a distribution of turbulent eddies over scale

$$N(l_n) \sim l_n^{-f}, \tag{20}$$

where

$$f = \ln q^d(1-\gamma)/\ln q. \tag{21}$$

Switching to a continuous description [$\ell_n \to \ell$ and $N(\ell_n) \to \rho(\ell)$], we obtain

$$\rho(l) \sim l^{-f-1}. \tag{22}$$

From Eqs. (8), (21), and (22) we obtain

$$D_\sigma = \ln q^d(1-\gamma)/\ln q, \tag{23}$$

i.e., in this model D_σ, q, and γ are related with one another. If D_σ and γ are fixed, then from the formula (23) we can find the scale fragmentation multiplicity of this cascade process in the form

$$q = (1-\gamma)^z. \tag{24}$$

Here

$$z = 1/(D_\sigma - d). \tag{25}$$

The formulas (24) and (25) could be helpful for determining the relation between the constant c in the Kolmogorov–Obukhov spectral law

$$E(k) = c\bar{\varepsilon}^{2/3}k^{-5/3} \tag{26}$$

and the intermittency coefficient γ. A relation between c and q was obtained in [13]:

$$c = q^{4/9}\frac{q^{2/3}}{(q^{2/3}-1)}3^{-2/3}. \tag{27}$$

Substituting into Eq. (27) the formulas (24) and (25), we find

$$c = (1-\gamma)^{4z/9}\frac{(1-\gamma)^{2z/3}}{[(1-\gamma)^{2z/3}-1]}3^{-2/3}. \tag{28}$$

We obtained previously $D_\sigma = 7/3$ with d = 3 for the case when the Kolmogorov–Obukhov spectral law is satisfied. Substituting these values into Eq. (28) we obtain

$$c(\gamma) = 3^{-2/3}/\gamma(1-\gamma)^{2/3}. \tag{29}$$

It is obvious that this formula is not applicable near the limits of the range of γ, i.e., near γ = 0 and γ = 1. However, at the minimum c = 1.48 at γ = 0.6, the function (29) increases in the interval 0.4 < γ < 0.8 by only 15%, i.e., in the interval 0.4 < γ < 0.8 the relation (29) may hold. Since c and γ are measurable quantities, Eq. (29) on this interval can be checked experimentally. We note that measurements of the dependence c(γ) in a boundary layer, wakes, and jets are not suitable for checking Eq. (29), since intermittency associated with the "outer" limit of turbulence is important in them (see, for example, [14]). Here measurements in the atmosphere and ocean would be very interesting for checking Eq. (29).

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HYDRAULIC RESISTANCE OF SWIRL CHAMBERS WITH A FLUID LAYER

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The industrial use of swirl chambers (SCs) is hampered by inadequate study of a number of questions, including determination of the hydraulic resistance ΔP_0 of SCs containing a fluid layer. This question has been discussed many times in the literature. Thus in [1-3] the following relations are proposed for SCs whose housing is stationary and in which gas spins the fluid layer:

$$\Delta P = 2\Delta P_0/(\rho'' W''^2) = 1 \quad (1)$$

where ρ" is the gas density and W" is the velocity of the gas between the guide vanes;

$$\Delta P = 0{,}8; \quad (2)$$

$$\Delta P = 23k \quad (3)$$

where $k = s\eta$, s is the relative flow area, $\eta = h/r_0$, and h and r_0 are the height and radius of the guide vanes.

The values of the hydraulic resistance calculated from Eqs. (1)-(3) cannot be compared with the experimental data of [1, 2], since the geometric dimensions of the SC are not given there. At the same time these values differ by several times from the experimental data of [3].

In [4] the following relations are given for calculating the hydraulic resistance of the fluid layer in application to SCs with a stationary housing and gas-spun fluid layer:

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